10.2 First Order Partial Derivatives

In this section, we will ...

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Question 1. For each of the following, compute the partial derivative specified using only the limit definition.

(a) $\frac{\partial f}{\partial x}$ for $f(x, y) = x^2 + 3xy + y^2$

(b) $\frac{\partial f}{\partial y}$ for $f(x, y) = x^2 + 3xy + y^2$

(c) $\frac{\partial f}{\partial x}$ for $f(x, y) = \frac{1}{1+x^2+y^2}$

Question 2. For each of the following, compute the partial derivative specified (you may use properties of derivatives)

(a) $\frac{\partial}{\partial x}(\sin(3x)\cos(3y))$

(b) $\frac{\partial}{\partial x}\left(x^8 e^{3y}\right)$

(c) $\frac{\partial}{\partial x}\left(e^{-1/(1-x^2-y^2)}\right)$

(d) $f_x(2,-2)$ where $f(x,y) = \frac{xy}{x-y}$.

(e) $\frac{\partial}{\partial y}\left(x^8 e^{3y}\right)$

(f) $\frac{\partial}{\partial y}\left(e^{-1/(1-x^2-y^2)}\right)$

Question 3. **Application:** The ideal gas law says, that for $n$mol of an "ideal gas", its temperature T (measured in Kelvin), pressure P (Nm^{-2}), and volume V (m^3) are related by the equation $PV = nRT$, where $R = 8.314\mathrm{Jmol}^{-1}\mathrm{K}^{-1}$. This gives us three functions, $P = P(T, V)$, $V = V(T, P)$, and $T = T(P, V)$. Use the ideal gas law to solve each of the following questions.

(a) Find $\frac{\partial P}{\partial V}$

(b) Find $\frac{\partial V}{\partial T}$

(c) Find $\frac{\partial T}{\partial P}$

(d) Use the last three parts to compute $\dfrac{\partial P}{\partial V}\dfrac{\partial V}{\partial T}\dfrac{\partial T}{\partial P}$.*

*This result is known as the "Cyclic Derivative Theorem"', and holds in general whenever you have an implicit equation of the form $f(x, y, z) = 0$.

Question 4. **Challenge Problem:** Consider the function

$$f(x,y) = \begin{cases} e^{\frac{-1}{1-\left(x^2+y^2\right)}} & 0 \leq x^2+y^2 < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Compute $f_x(x,y)$ and $f_y(x,y)$ for points inside the open unit disk $0 \leq x^2+y^2 < 1$.

(b) Compute $f_x(x,y)$ and $f_y(x,y)$ for points outside the open unit disk, that is, when $x^2+y^2 > 1$.

(c) At the boundary of these two regions is the unit circle $x^2+y^2=1$. Use your answers to parts (a) and (b) to show that $f_x(x,y)$ and $f_y(x,y)$ are continuous at any point on the unit circle. (Hint: What value do $f_x(x,y)$ and $f_y(x,y)$ approach as (x,y) approach a boundary point from the inside of the unit disk?)