Final Exam Review I

Big Picture of Calc III:

① Four types of functions:

- Single-variable funs: 1 in, 1 out (Calc I/II) $y=f(x)$
- multivariable funcs:

  ⓐ functions of >1 var: $z=f(x,y)$ $w=f(x,y,z)$ etc.

  ⓑ Parametric funcs: 1 input >1 output $\vec{r}(t)$

  ⓒ Vector fields: >1 input, >1 output $\vec{F}(x,y,z)$

(2) Working in 3D "the toolbox"

↳ Vectors, geometry, types of surfaces / regions,

parametric surfaces, etc.

(3) Derivatives & types of derivatives:

↳ ordinary derivative from Calc 1

Partial derivs (Ch 10)

derivs of Param. funcs (Ch 9)

gradients, curls, divergences

Lagrange multipliers,

optimization,

tangent lines / planes

chain rule, etc.

(4) Integrals, oh gosh, so many types of integrals

Double, triple, Line, Flux

arclength, Big theorems: Green's Thm, Stokes' thm,

Div. thm,

Volume, Surface area, area, ...

(5) Vector fields & related theorems: (Ch 12)

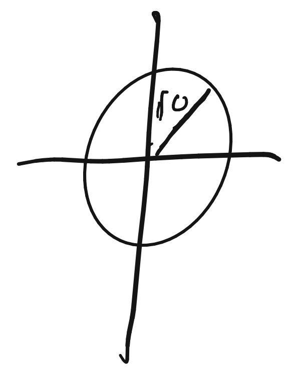
P.I. v-field, potential functions, geom. of v-fields...

Review of Lagrange Multipliers:

Target func $f(x,y)$ ← try to optimize this

Constraint func $g(x,y)$ ← Subject to this constraint.

$$g(x,y) = x^2 + y^2 = 100$$



$$f(x,y) = x + 3y$$

$$\begin{cases} \vec{\nabla} f = \lambda \vec{\nabla} g \\ g(x,y) = 100 \end{cases}$$

$\vec{\nabla} f = \langle 1, 3 \rangle$

$\vec{\nabla} g = \langle 2x, 2y \rangle$

$\Rightarrow$

$1 = 2\lambda x$

$3 = 2\lambda y$

$x^2 + y^2 = 100$

$x = \frac{1}{2\lambda}$

$y = \frac{3}{2\lambda}$

$\left(\frac{1}{2\lambda}\right)^2 + \left(\frac{3}{2\lambda}\right)^2 = 100$

$\frac{1}{4\lambda^2} + \frac{9}{4\lambda^2} = 100$

$= \frac{10}{4\lambda^2} = 100$

$$10 = 400\lambda^2$$

$$1 = 40\lambda^2$$

$$\lambda^2 = \frac{1}{40} \Rightarrow \lambda = \pm\sqrt{\frac{1}{40}}$$

$$x = \frac{1}{2\lambda} \qquad\qquad x = \pm \tfrac{1}{2}\cdot\sqrt{1/40} = \pm\frac{\sqrt{40}}{2}$$

$$y = \frac{3}{2\lambda} \Rightarrow$$

$$y = \pm\frac{3\sqrt{40}}{2}$$

4 Crit points: $\left(+\frac{\sqrt{40}}{2}, \frac{3\sqrt{40}}{2}\right)$, $\left(-\frac{\sqrt{40}}{2}, \frac{3\sqrt{40}}{2}\right)$

max

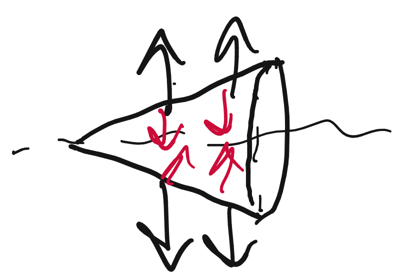
$\left(+\frac{\sqrt{40}}{2}, -\frac{3\sqrt{40}}{2}\right)$, $\left(-\frac{\sqrt{40}}{2}, -\frac{3\sqrt{40}}{2}\right)$ min.

$f = x + 3y$

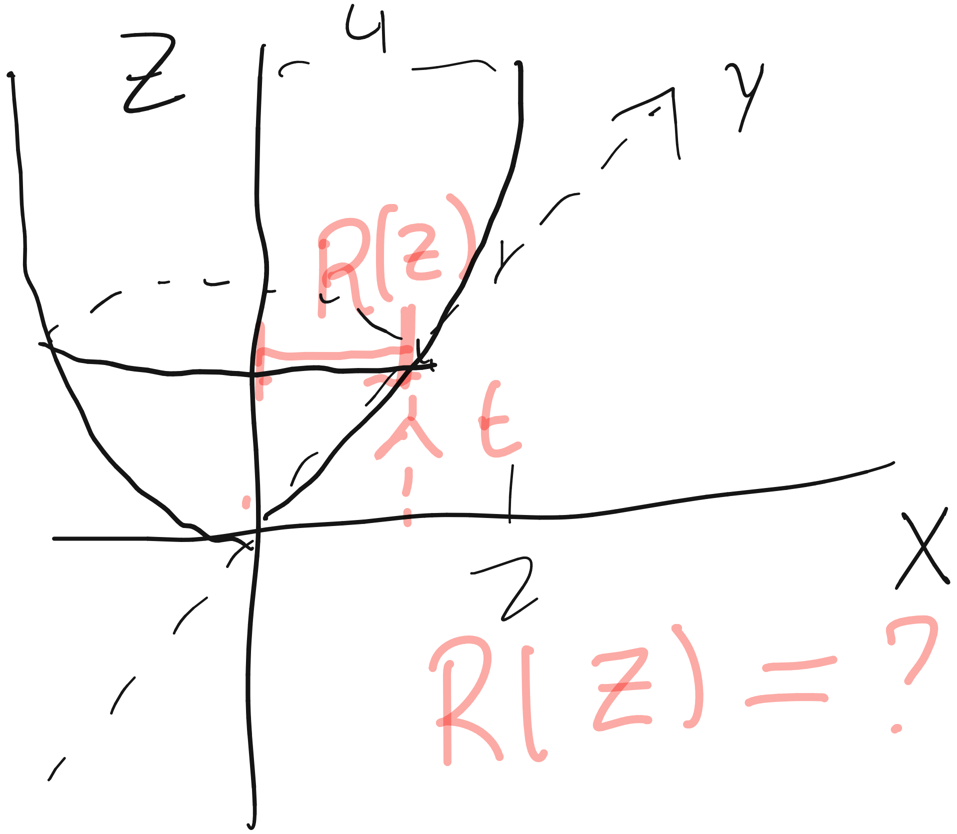
Big Idea w/ Param. Surfaces:

1. "Std trick"
2. Surfaces of revolution → "Spicy Cylinders"!

→ $z = f(x,y) \Rightarrow \langle s, t, f(s,t) \rangle$

→  $\times$ $\langle t, \ t \cdot \sin(s), \ t \cdot \cos(s) \rangle$

$n = r_s \times r_t = \ldots$



$R(z) = ?$

Spin $z = x^2$ about z-axis

for $0 \le x \le 2$.

$\Downarrow$

$0 \le z \le 4$

$\left[R(z) = \sqrt{z} \right]$ radius func.

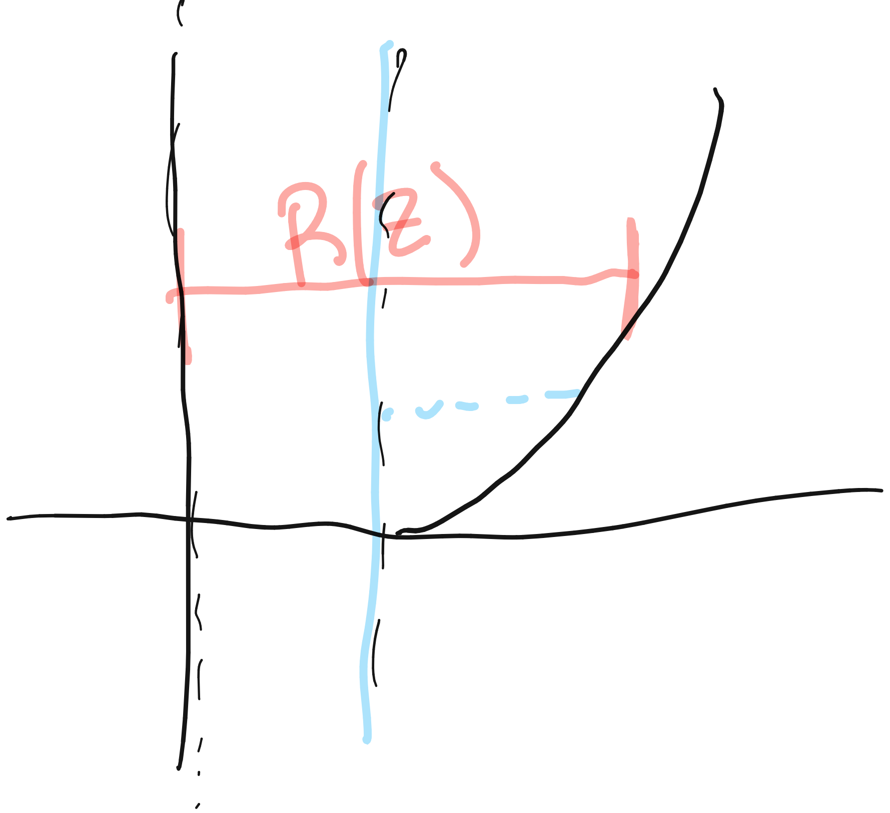
$\sqrt{t}\,((\cos(s))^2$

$\langle \sqrt{t}\cos s, \quad \sqrt{t}\sin s, \quad t \rangle$

$0 \le t \le 4$

$[\, 0 \le s \le 2\pi \,]$

"$(2, 0, z)$"



$$z = (x-2)^2 \qquad (2 \le x \le 4)$$

$$x(z) = (\sqrt{z}) + 2$$

$$R(t) = (\sqrt{t}) + 2$$

$$= 2 + \sqrt{t}$$