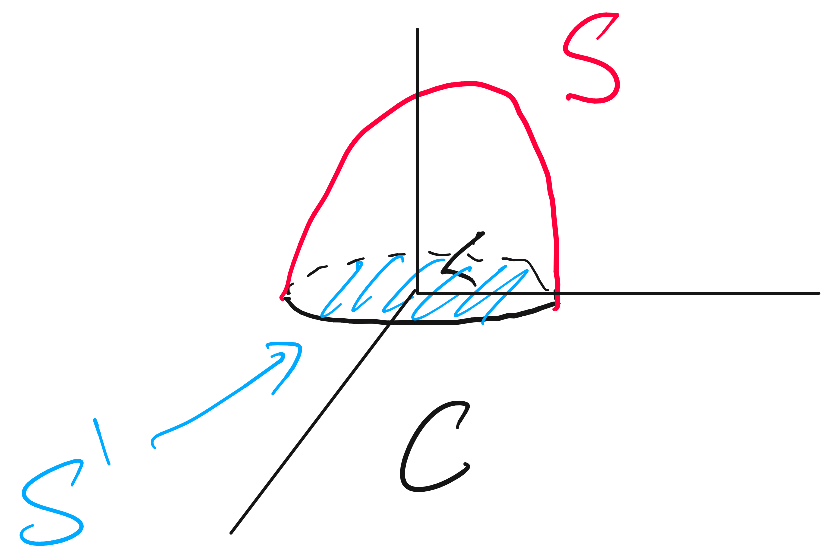
Additional Ex of Stokes' Theorem

$F = \langle z, x, y \rangle$



C is circle of radius a

in xy-plane ctd @ origin.

oriented CCW

S': disk of radius a centered @ origin contained in x-y plane

oriented upwards.

Last time: $\oint_C \vec{F}\cdot d\vec{r} = a^2\pi$, $\iint_S \text{Curl}(F)\cdot\vec{n}\, dA = a^2\pi$

Need to parametrize S':

$$\vec{r}(s,t) = \langle s\cdot\cos t,\ s\sin t,\ 0\rangle \qquad 0\le s\le a$$
$$0 \le t \le 2\pi.$$

Want to compute $\iint_{S'} \text{Curl}(\vec{F})\cdot\vec{n}\, dA$

last time: $\text{Curl}(\vec{F}) = \langle 1, 1, 1\rangle$

$\vec{r}_s = \langle \cos t, \sin t, 0\rangle$

$\vec{r}_t = \langle -s\sin t, s\cos t, 0\rangle$

$\vec{r}_s \times \vec{r}_t = \langle 0, 0, s\rangle = \vec{n}$

$$\iint_{S'} \text{Curl}(\vec{F}) \cdot \vec{n}\, dA = \iint_{S'} \langle 1,1,1\rangle \cdot \langle 0,0,s\rangle \, dA$$

$$= \int_0^{2\pi}\int_0^{a} s \, ds\, dt = 2\pi \cdot \frac{1}{2}a^2 = \boxed{a^2\pi}$$

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_{S'} \text{Curl}(\vec{F}) \cdot \vec{n}\, dA \quad \boxed{\checkmark}$$