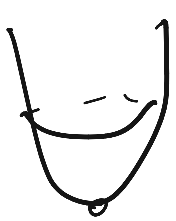
Today is cutoff for Exam 1 material:

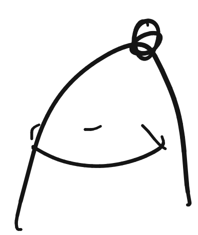
Exam 1: 9.1 – 9.5, 10.1 – 10.7

Yesterday: $f(x,y)$ function $p = (a,b)$ is a critical point if $\vec{\nabla} f(p) = \vec{0}$

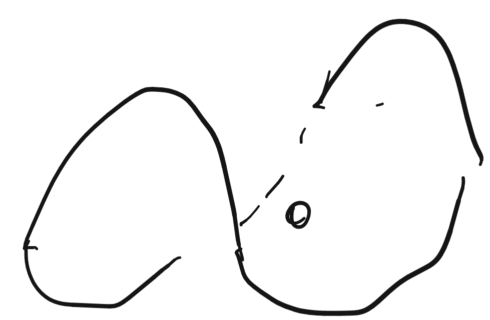
Crit. points come in 3 flavors:



local mins



local maxes



Saddle points.

<u>Today</u>: Analogue of the Calc 1 <u>2^{nd} deriv test</u>:

In Calc 1 the 2^{nd} deriv test is as follows:

$f(x)$ function p is crit. point $(f'(p)=0)$

Then: if $f''(p) > 0 \to$ local min

$f''(p) < 0 \to$ local max

$f''(p) = 0 \to$ test is inconclusive

<u>Def'n</u> Let $f(x,y)$ be a function st

f_{xx}, f_{yy}, f_{xy} exist.

The <u>discriminant</u> (Hessian determinant) of f

is the quantity $D = f_{xx} f_{yy} - (f_{xy})^2$

Here, D will play the same role as $f''(p)$

does in the Calc I 2^{nd} deriv test.

Theorem (2nd Deriv test)

Let $f(x,y)$ a func of two variables,

$p = (a,b)$ a critical point of f.

Then: ① If $D(p) > 0$ and Either $f_{xx}(p)$ or $f_{yy}(p) > 0$

then p is a local min.

② If $D(p) > 0$ and Either $f_{xx}(p) < 0$ or $f_{yy}(p) < 0$

then p is a local max.

③ If $D(p) < 0$ then p is saddle point.

④ If $D(p) = 0$, the test is inconclusive.

Ex $f(x,y) = 3x^3 + y^2 - 9x + 4y.$

Yesterday, we saw that this func has two

Crit pts: $(1,-2), (-1,-2)$

Goal: Classify these crit pts.

$f_{xx} = 18x$ $\quad$ $f_{yy} = 2$

$f_{xy} = 0$

So $D(x,y) = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$D = 36x - 0^2 = 36x.$

<u>Test:</u> $D(1,-2) = 36 \cdot 1 = 36. > 0$

need to look @ sign of f_{xx} or f_{yy}.

$f_{yy}(1,-2) = 2 > 0 \implies$ <u>local min</u>

(1,-2) is a local min of $f(x,y)$

<u>Test:</u> $D(-1,-2) = 36 \cdot (-1) = -36 < 0$

$\hookrightarrow$ $\boxed{(-1,-2) \text{ is saddle point.}}$

Ex A closed rectangular box has volume 30 cm^3.

Goal: Find side lengths of Edges giving min'l Surface area.

$V = lwh = 30$. Let $h = 30/lw$.

$S = 2lw + 2lh + 2hw$.

$= 2lw + \frac{60}{w} + \frac{60}{l}$

Aside: What we want to do is find all crit points of S and look for all local mins.

$$\vec{\nabla} S = \langle S_l, S_w \rangle$$

$$= \left\langle 2w - \frac{60}{l^2},\ 2l - 60/w^2 \right\rangle = \vec{0}$$

$$\begin{cases} 2w = 60/l^2 \\ 2l = 60/w^2. \end{cases}$$

$$wl^2 = w^2 l \Rightarrow \boxed{w = l.}$$

divide top eq'n by bottom eq'n:

$$\frac{2w}{2l} = \frac{60/l^2}{60/w^2} \Rightarrow \boxed{\frac{w}{l} = \frac{w^2}{l^2}}$$

Only happens when $\boxed{w = l.}$

$2l = 60/l^2 \Rightarrow 2l^3 = 60 \Rightarrow \boxed{l = \sqrt[3]{30}}$

$\Downarrow$

$30 = \dfrac{30^{3/3}}{30^{2/3}} = 30^{1/3}$ $\boxed{w = \sqrt[3]{30}}$

$\Downarrow$

$h = \dfrac{30}{(\sqrt[3]{30})^2} = \sqrt[3]{30}$ $\boxed{h = \sqrt[3]{30}}$

So $(\sqrt[3]{30}, \sqrt[3]{30})$ is a crit. point of $S(l,w)$.

Check: Is it a local min?

$S_{\ell\ell} = \frac{120}{\ell^3}$ $S_{ww} = \frac{120}{w^3}$

$S_{\ell w} = 2.$ $S_{\ell\ell}(\sqrt[3]{30}, \sqrt[3]{30}) > 0 \implies \boxed{\text{local min!}}$

$$D(\sqrt[3]{30}, \sqrt[3]{30}) = \frac{120}{(\sqrt[3]{30})^3} \cdot \frac{120}{(\sqrt[3]{30})^3} - (2)^2$$

$$= \frac{120}{30} \cdot \frac{120}{30} - 4$$

$$= 4 \cdot 4 - 4 = 12 > 0.$$

In summary, we found that the optimal side lengths are $l = w = h = \sqrt[3]{30}$ cm and these side lengths minimize surface area of the box.