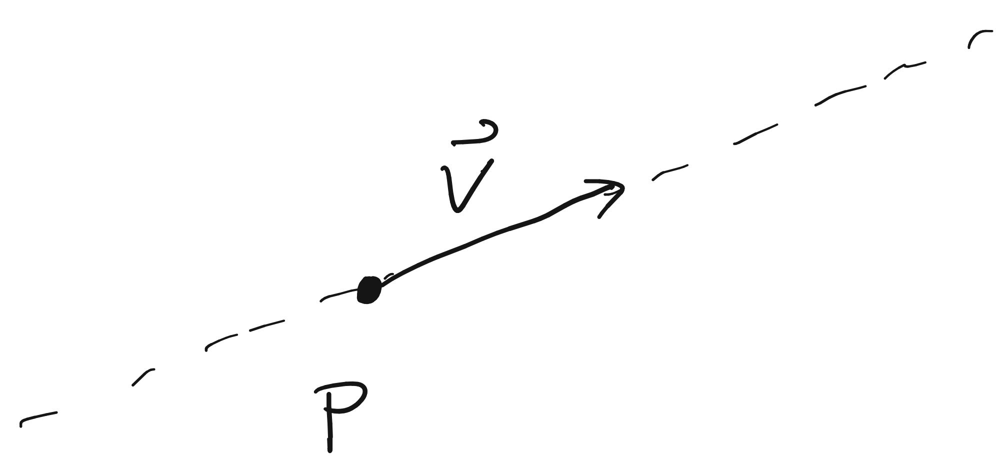
§9.5

## Last time:

**Line:** Starting point $P = (P_1, P_2, P_3)$




direction vector $\vec{v} = \langle v_1, v_2, v_3 \rangle$

$$\ell(t) = \vec{OP} + t\vec{v} = \underline{\langle P_1 + tv_1,\ P_2 + tv_2,\ P_3 + tv_3 \rangle}.$$

Origin

**Plane:** normal vector $\vec{n} = \langle a, b, c \rangle$

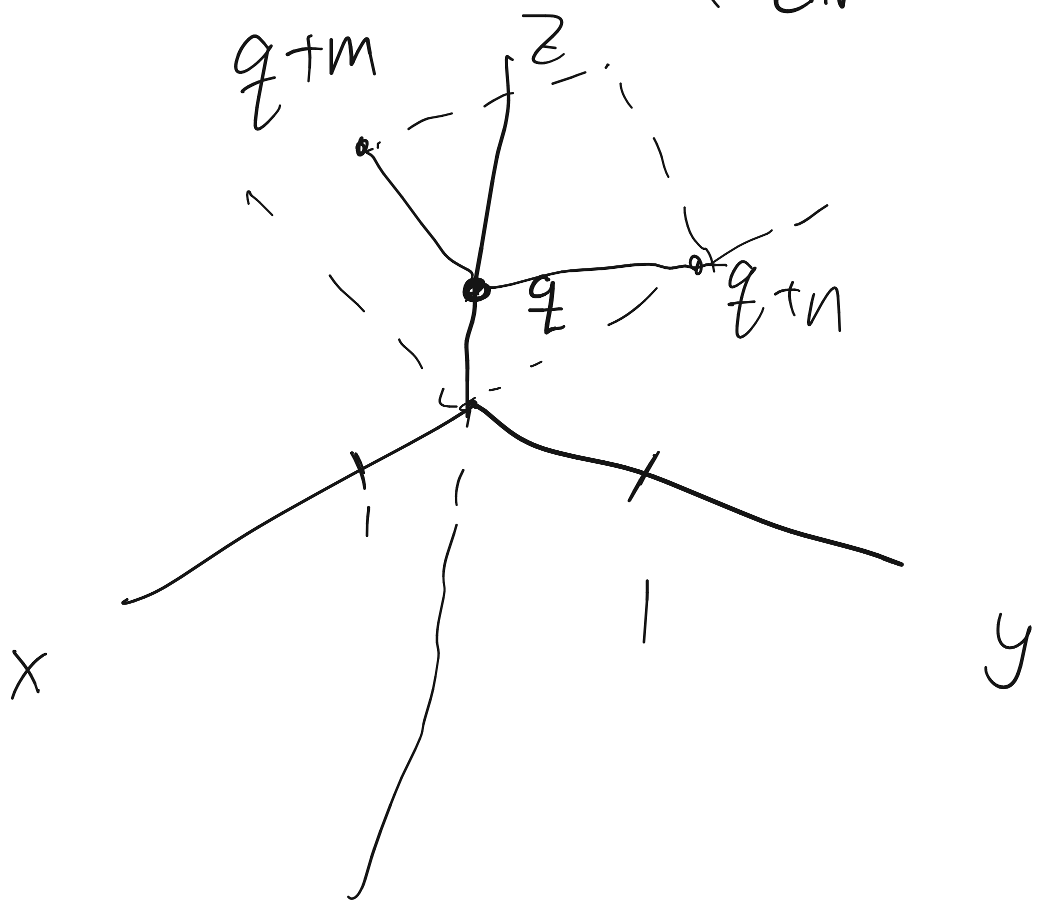
point $P_0 = (x_0, y_0, z_0)$, $P = (x, y, z)$

$$\boxed{a(x - x_0) + b(y - y_0) + c(z - z_0) = 0} \quad \text{or} \quad \underline{\vec{n} \cdot \vec{P_0P} = 0}$$

If you rearrange eqn for $z = f(x,y)$

$$z = mx + ny + q$$

Slope in x dir. — Slope in y dir — z-intercept.

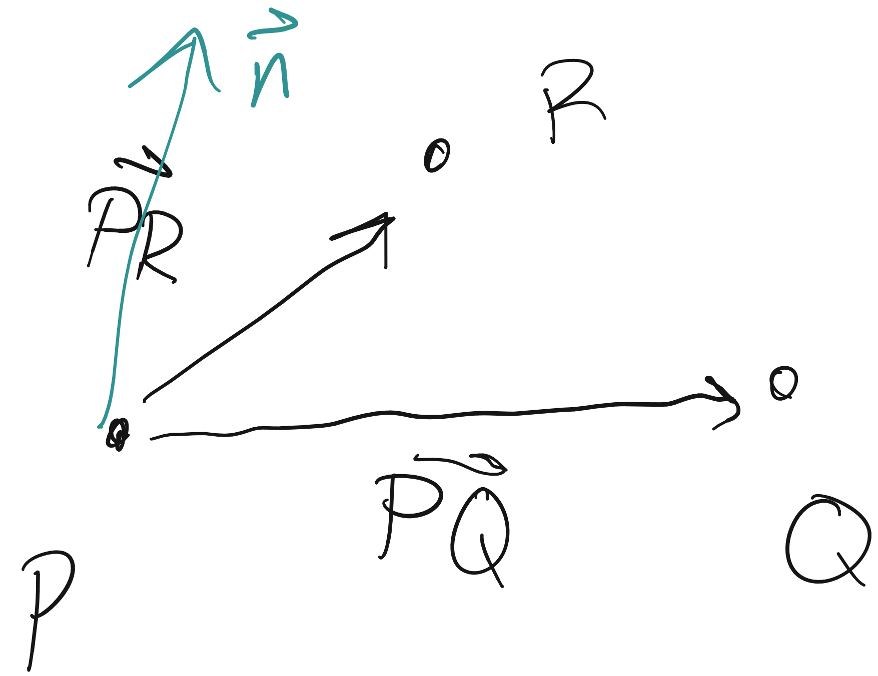



Any 3 (non-colinear) points in 3-space form a plane.

"general position"

**Method** Eqn of a plane through 3 points.

$P, Q, R$ points in $\mathbb{R}^3$.




$\vec{n} = \vec{PQ} \times \vec{PR}$ normal vector.

Use $P$ as our "base point".

if $n = \langle a, b, c \rangle$ $P = (p_1, p_2, p_3)$

$$\boxed{a(x - p_1) + b(y - p_2) + c(z - p_3) = 0}$$

Ex $P = (1, 2, -1)$

$Q = (1, 0, -1)$

$R = (0, 1, 3)$

① Find $\vec{PQ}$, $\vec{PR}$

② $\vec{n} = \vec{PQ} \times \vec{PR}$

③ Combine ingredients to get Eqn of plane.

$\vec{PQ} = \langle 0, -2, 0 \rangle$

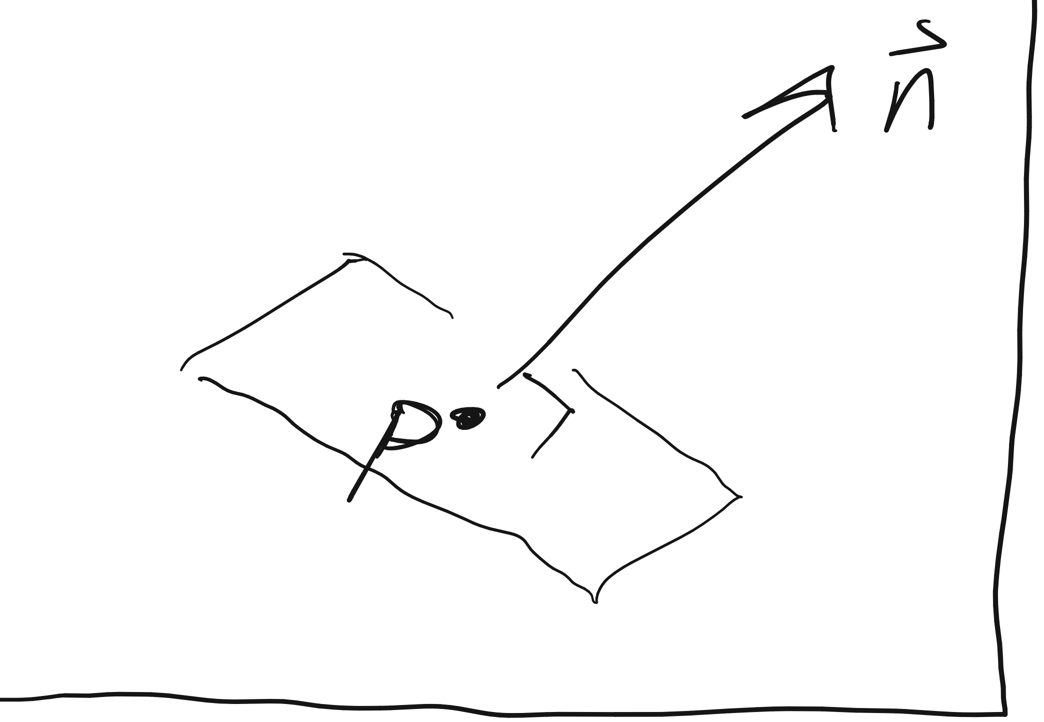
$\vec{PR} = \langle -1, -1, 4 \rangle$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & -2 & 0 \\ -1 & -1 & 4 \end{vmatrix} =$$

$$\hat{\imath}(-8+0) - \hat{\jmath}(0-0) + \hat{k}(0-2)$$

$$= -8\hat{\imath} - 2\hat{k} = \vec{n}$$

$$-8(x-1) + 0(y-2) - 2(z+1) = 0$$




| y\x | 2 | 3 |
|---|---|---|
| 1 | 17 | 22 |
| 0 | 16 | 21 |
| −1 | 15 | 20 |

$\frac{\Delta z}{\Delta y} = 1$

$\frac{22-21}{1-0} = 1$

$\vec{n}$

① fill out table

② find the eqn. represents a $\boxed{\text{linear function.}}$

$z = mx + ny + q$

$m = 5$, $n = 1$

$\frac{20-15}{3-2} = \frac{\Delta z}{\Delta x} = \frac{5}{1} = 5$

$21 = 5 \cdot 3 + 1 \cdot 0 + q$

$\boxed{z = 5x + 1y + 6.}$ $\quad 21 = 15 + q \Rightarrow q = 6$

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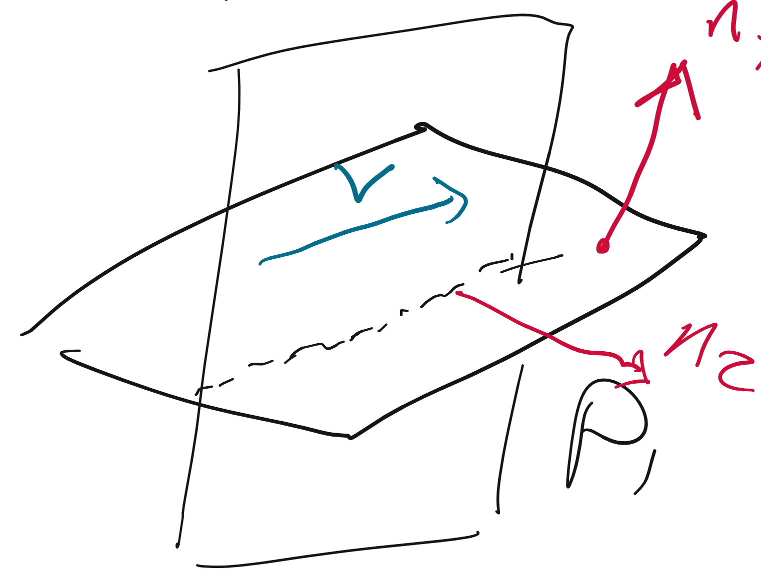
Ex two planes $P_1$, $P_2$ w/ normal vectors $\vec{n}_1$, $\vec{n}_2$ respectively.

Find a vector $V$ that's parallel to the intersection of $P_1$ and $P_2$.

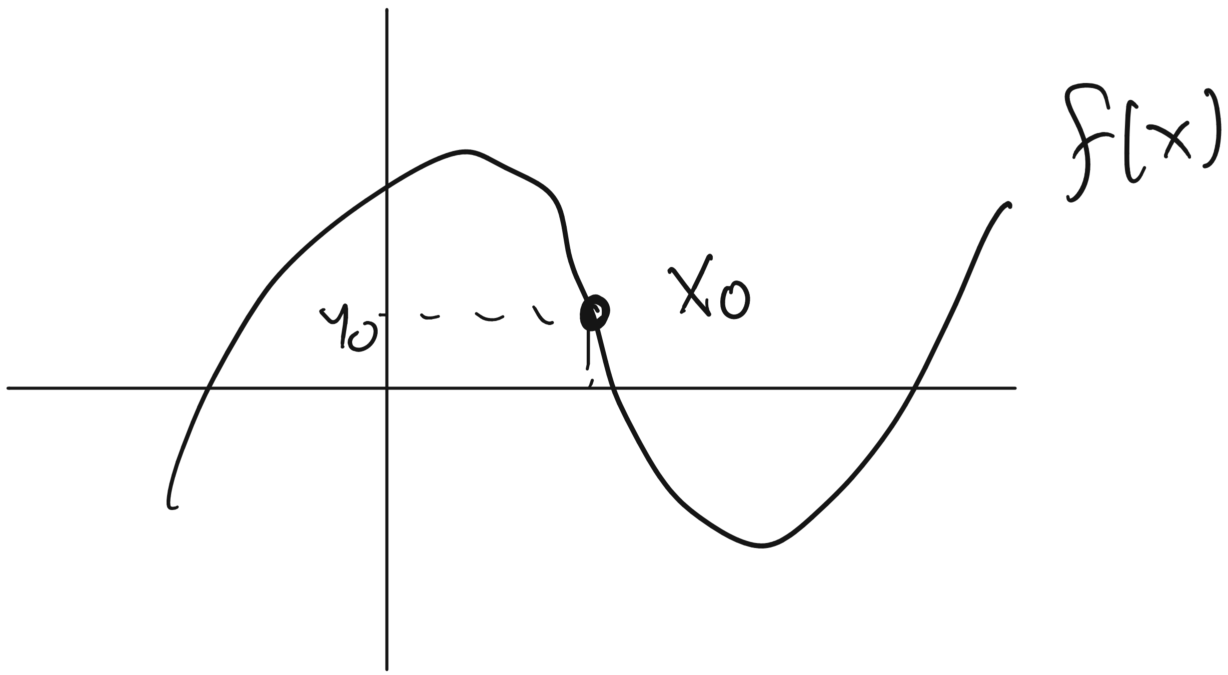



Trick: $V$ should be pll to $\vec{n}_1 \times \vec{n}_2$

# Hint for WW 9.5 #2

Given a contour diagram Pick 3 points w/ different $(x, y, z)$ coords & use that to find the Eqn of a plane

# Limits




We say that $\lim_{x \to x_0} f(x) = y_0$ if

① the limit exists. ie $\lim_{x \to x_0^+}$, $\lim_{x \to x^-}$ exist & agree

Idea: we need to approach $x_0$ from all feasible dirs.

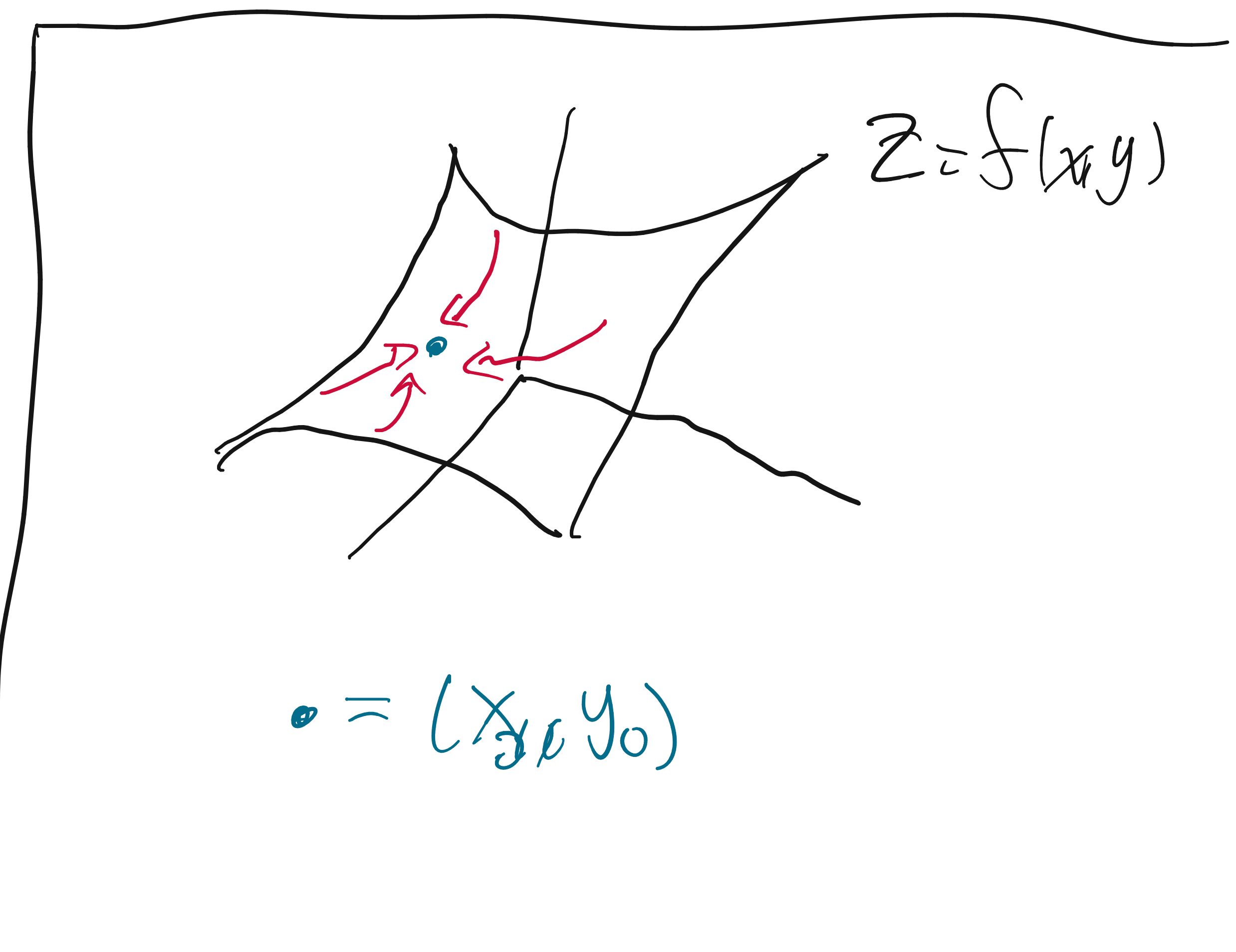
**Def** given a func. $z = f(x,y)$ we say that

$f(x,y)$ has limit $L$ as $(x,y) \to (x_0,y_0)$ if

we can make $f(x,y)$ as close to $L$ as we want by taking $(x,y)$ sufficiently close to (but not equal to) $(x_0,y_0)$.

If we can do this, we write

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$$



Ex $g(x,y) = \dfrac{x^2y}{x^4+y^2}$

Investigate

$$\lim_{(x,y)\to(0,0)} g(x,y)$$

① take the limit along the $y=0$ slice.

$$g(x,0) = \frac{x^2\cdot 0}{x^4+0} = 0$$

$$\lim_{x\to 0} g(x,0) = 0.$$

② try $x=0$ slice. $g(0,y)=0$, so $\lim_{y\to 0} g(0,y) = 0$.

③ let's try coming into $(0,0)$ along the curve

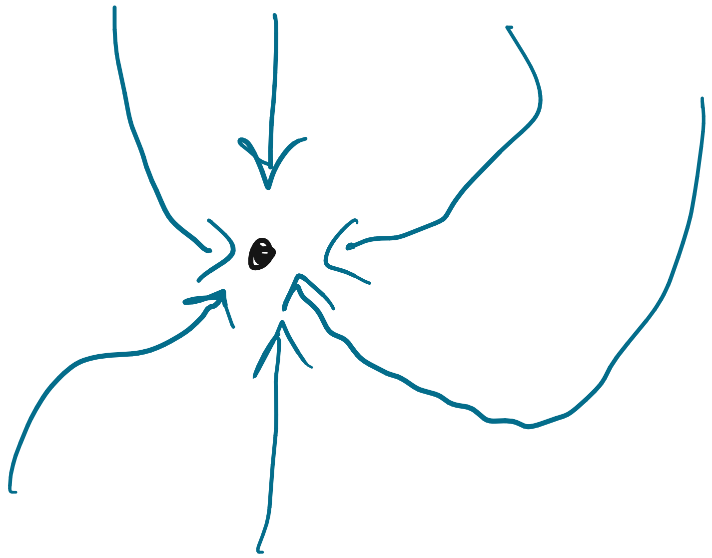
$y = x^2$.

$$g(x, x^2) = \frac{x^2 \cdot x^2}{x^4 + (x^2)^2} = \frac{x^4}{2x^4} = \frac{1}{2}$$

So $\lim\limits_{x \to 0} g(x, x^2) = \frac{1}{2}$. (!)

What does this mean?

↳ $\lim\limits_{(x,y) \to (0,0)} g(x,y)$ DNE b/c we found two paths w/ different limit values!



"Picture of checking all possible directions".

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A func. $f(x,y)$ is continuous (cts.) at $(a,b)$ if

① $\lim_{(x,y)\to(a,b)} f(x,y)$ exists,

② $f(a,b)$ is defined, and

③ $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$

Ex We can ask the following:

$$f(x,y) = \begin{cases} x^3+y^3+5 ; & (x,y) \neq (0,0) \\ c ; & (x,y) = (0,0). \end{cases}$$

Q What value of $c$ makes $f(x,y)$ continuous @ $(0,0)$?

A $c=5$.

↳ why? $\lim_{(x,y)\to(0,0)} f(x,y) = 5$

So if $f$ were to be cts @ $(0,0)$ we must have

$$f(0,0) = 5.$$