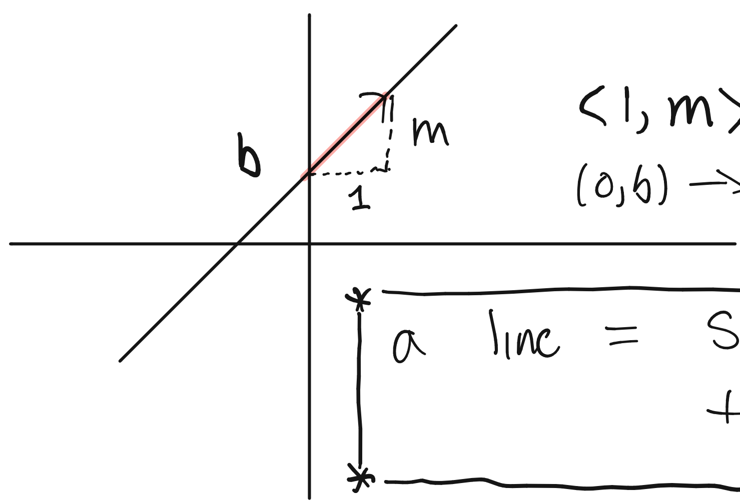
§ 9.5: Lines & Planes in 3-Space

<u>Review</u> a line in 2D is a a function

$$y = m(x - x_0) + y_0$$

(x_0, y_0) Starting Point $\quad m = \frac{\Delta y}{\Delta x}$ Slope



$\langle 1, m \rangle \to$

$(0, b) \to (1, b+m)$

a line = Starting point + direction

Idea: Point $P = (x_0, y_0, z_0)$

direction of travel $\vec{V} = \langle a, b, c \rangle$

The (vector form) of the line starting @ P in direction $\vec{v}$ is

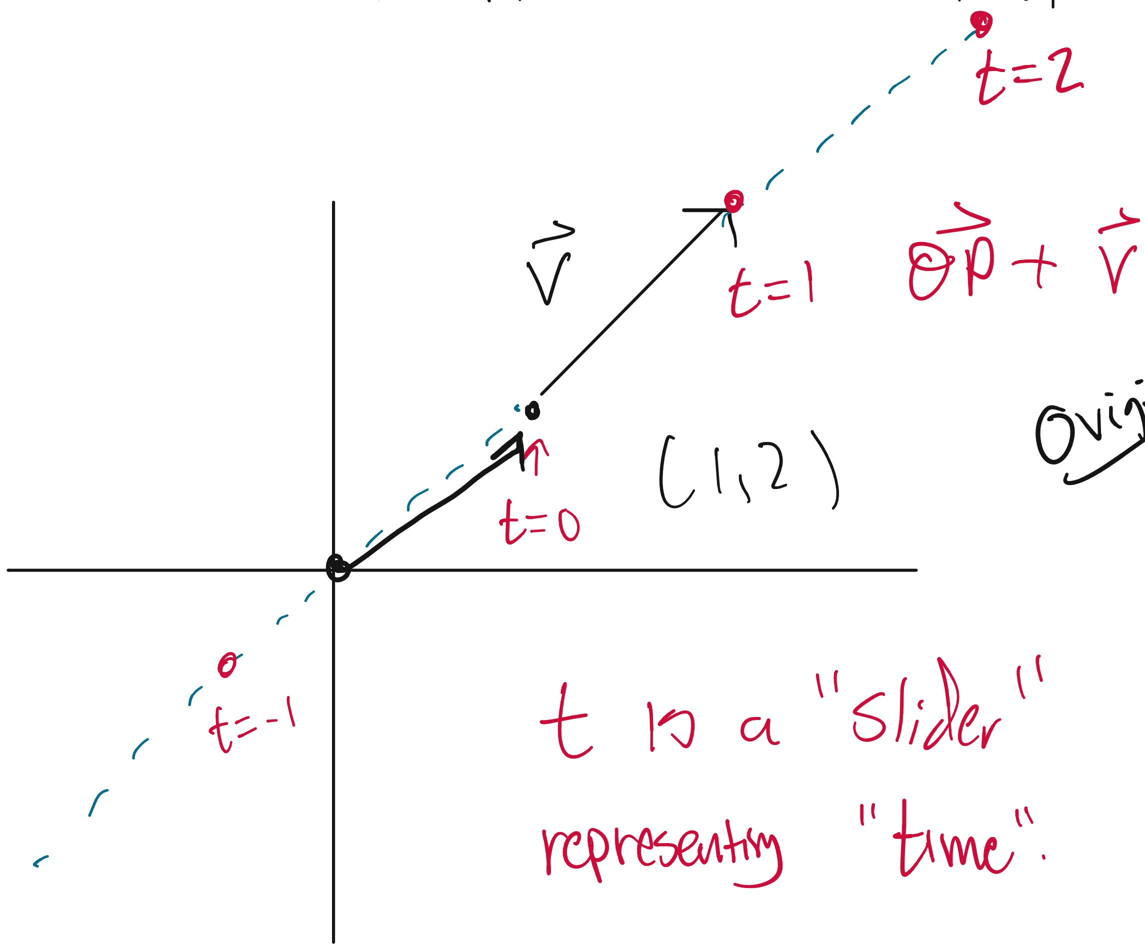
$$\vec{r}(t) = \overrightarrow{OP} + t\vec{v} \qquad \Theta \qquad H$$

O = origin

Ex

Starting point: $(1,2)$

Initial direction $\langle 3, 4 \rangle$



t is a "slider" representing "time".

origin $\rightarrow$ $\vec{OP} = \langle 1,2 \rangle$

$\vec{v} = \langle 3,4 \rangle$

$$\vec{r}(t) = \langle 1,2 \rangle + t\langle 3,4 \rangle.$$

$$= \langle 1+3t,\ 2+4t \rangle.$$

Ex find Eqn of a line from

$P = (2,3,4)$ to $Q = (5,9,5)$.

two ingredients: <u>initial point</u>: $P = (2,3,4)$

<u>direction vector</u>: $\overrightarrow{PQ} = \langle 3,6,1 \rangle$

$$\vec{r}(t) = \overrightarrow{OP} + t\vec{v} = \langle 2,3,4 \rangle + t\langle 3,6,1 \rangle$$

$$= \langle 2+3t,\ 3+6t,\ 4+t \rangle$$

Parametric Equations.

$$r(t) = \begin{cases} x(t) = 2+3t \\ y(t) = 3+6t \\ z(t) = 4+t \end{cases}$$

$\leftarrow$ Parametric form of the line.

Ex $P = (1, 2, -1)$ $\quad$ $Q = (-2, 1, -2)$

$\mathcal{L}$ is the line from P to Q.

① find the Parametric form of $\vec{\mathcal{L}}(t)$.

$$\vec{PQ} = \langle -3, -1, -1 \rangle$$

$$\ell(t) = \vec{OP} + t\,\vec{PQ}$$

$$= \langle 1, 2, -1 \rangle + t\langle -3, -1, -1 \rangle$$

$$= \begin{cases} x(t) = 1 - 3t \\ y(t) = 2 - t \\ z(t) = -1 - t. \end{cases}$$

②: K be the line w/ parametric form

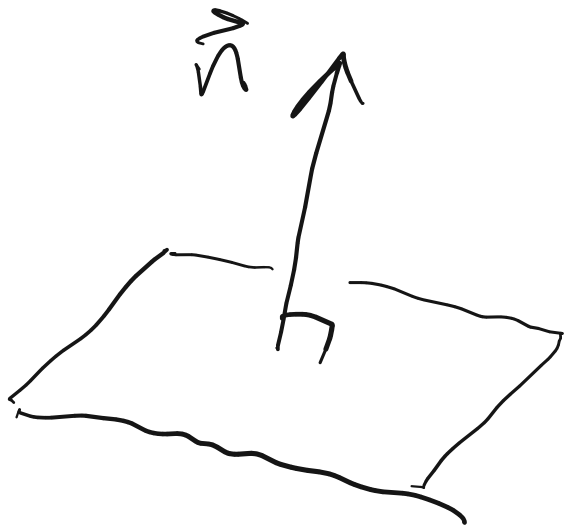
$$K(s) = \begin{cases} x(s) = 11+4s \\ y(s) = 1-3s \\ z(s) = 3+2s \end{cases}.$$

Find direction vector of K

$$\vec{v} = \langle 4, -3, 2 \rangle.$$

Planes in 3D

A **plane** is the set of all points $\vec{p}$ perpendicular to a given normal vector $\vec{n}$.



① Vector form:

$P_0 = (x_0, y_0, z_0)$

$P = (x, y, z)$

$$\vec{n} \cdot \overrightarrow{P_0P} = 0.$$

② Scalar form.

$\vec{n} = \langle a, b, c \rangle$ normal vector

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

(x_0, y_0, z_0) is a point on plane

$\underline{2^a}$ Scalar form pt 2:

$$ax + by + cz = d$$

is also a plane.

$$2(x-0) + -1(y-2) + 1(z-4) = 0$$

$\underline{\text{Exs}}$ ① find a plane cont. the point $P_0 = (\overset{x_0}{0}, \overset{y_0}{2}, \overset{z_0}{4})$

w/ $\vec{n} = \langle \overset{a}{2}, \overset{b}{-1}, \overset{c}{1} \rangle$

② Q: is the point $(2,0,2)$ on this plane?

$$2(2-0) + -1(0-2) + 1(2-4) \overset{?}{=} 0$$

$$4 + 2 - 2 = 4 \neq 0$$

So the point $(2,0,2)$ is not on this plane.

Note: two planes P_1, P_2 are parallel if their normal vectors are parallel.

ie if $\vec{n}_1 \parallel \vec{n}_2$

$\vec{n} = \langle \overset{a}{2}, \overset{b}{-1}, \overset{c}{1} \rangle$

pll normal vectors

find a plane pll to the plane above goes through the point $(3,0,4)$

$$2(x-3)-(y-0)+(z-4)=0.$$