Name: ______________________________

Circle the name of your instructor:

LUTZ	PETERSON	TRUE	KERIAN	TRUE	TRAGERSER	TRUE
8:30 am	9:30 am	10:30 am	11:30 am	12:30 pm	12:30 pm	6:30 pm

Instructions

- There are **16** questions on **9** pages (including this cover sheet).
- No books or other notes are allowed.
- You may use a calculator.
- Turn off all communication devices.
- **Show all your work** and explain your answers. Unsupported answers will receive **little credit**.
- If specified, use the method required by each problem. Alternate methods will not receive full credit.
- You have **2 hours** to complete the exam.

Good luck !

Question	Out of	Score
1	12	
2	12	
3	12	
4	12	
5	13	
6	13	
7	12	
8	12	
9	13	
10	12	
11	13	
12	12	
13	13	
14	13	
15	14	
16	12	
TOTAL	200	

1. (12 points) Given the vectors $\vec{u} = 3\vec{i} + 4\vec{k} = < 3, 0, 4 >$ and $\vec{v} = \vec{i} + 2\vec{j} - 2\vec{k} = < 1, 2, -2 >$.

   (a) Find $\|2\vec{u} - 3\vec{v}\|$.

   (b) Find the vector that points in the same direction as $\vec{u}$ and has length 4.

   (c) Find the area of the triangle determined by $\vec{u}$ and $\vec{v}$.

2. (12 points)

   (a) Find a vector normal to the surface $z^2 - 3x^2 + 4y^2 = 9$ at the point $(1, 1, 4)$.

   (b) Find an equation of the plane tangent to the above surface at $(1, 1, 4)$.

3. (12 points) A cylindrical drill with radius 1 is used to bore a hole through the center of a sphere of radius 2. Find a triple iterated integral in spherical coordinates (don't evaluate) that gives the volume of the ring-shaped solid that remains.

4. (12 points) Find an equation for the tangent plane to the surface $z = f(x,y) = x^2y - 3xy^2$ corresponding to the point $(1,2)$.

5. (13 points)

   (a) For the function $f(x,y) = x^2 - y^2$, compute the directional derivative at $P = (1,2)$ in the direction from $P = (1,2)$ to $Q = (4,-2)$.

   (b) What is the value of the largest directional derivative of $f(x,y) = x^2 - y^2$ at the point $(1,2)$.

6. (13 points)

   (a) Find the **critical points** of $f(x,y) = \frac{1}{3}x^3 - 16x + 3y^2 - 2y$.

   (b) Use the **Second Derivative Test** to classify each of the critical points as a local maximum, local minimum, or saddle point.

7. (12 points) Given $z = x\sin y + y\sin x$, $x = 2t$, $y = 1 - t^2$, use the **chain rule** to find $\frac{dz}{dt}$.

8. (12 points) Given that the position vector of a particle of mass is given by

$$\vec{r}(t) =< t^2 - 4t, 6t^2 - 18t + 12 > .$$

Find the velocity vector, acceleration vector, and speed of the particle of mass **all** at time $t = 1$.

9. (13 points) Sketch the region of the integration for the double integral $\int_0^3 \int_0^{9-x^2} f(x,y)\, dydx$ in the xy-plane. Clearly shade this region. Rewrite the integral reversing the order the integration.

10. (12 points) Use **Lagrange multipliers** to find the point(s) on the ellipse $\frac{x^2}{2} + \frac{y^2}{8} = 1$ where the maximum and minimum values of $f(x,y) = 2xy$ occur.

11. (13 points) Let $\vec{F}(x,y) = 2xy\vec{i} + x^2\vec{j} = < 2xy, x^2 >$ and let C be the line segment from $(0,1)$ to $(2,2)$.
(a) Find a parametrization of the curve C.

(b) Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$.

12. (12 points) Find the upward flux of the vector field

$$\vec{F}(x,y,z) = (z - x^2)\vec{i} + (y+1)\vec{j} + 2y\vec{k} = < z - x^2, y+1, 2y >$$

through the part of the surface $z = f(x,y) = x^2 + y^2$ above the rectangle $0 \le x \le 2$, $0 \le y \le 1$.

13. (13 points) Let Q be the region bounded below by the cone $z = \sqrt{x^2+y^2}$ and bounded above by the sphere $x^2+y^2+z^2=4$. Let S be the bounding surface of the region Q. Use the **Diverence Theorem** to find the outward flux of the vector field $\vec{F} = (xz^2)\vec{i} + \frac{1}{3}y^3\vec{j} + (x^2z+2xy)\vec{k} =< xz^2, \frac{1}{3}y^3, x^2z+2xy >$ through the surface S.

14. (13 points) Use **Stoke's Theorem** to find a **double integral** that gives the circulation of the vector field $\vec{F}(x,y,z) = z^2\vec{i} + 3xy\vec{j} + 5yz\vec{k} =< z^2, 3xy, 5yz >$ around the triangle from $P = (2,0,0)$, to $Q = (0,2,0)$, to $R = (0,0,4)$ and back to P (this triangle lies in the plane $2x+2y+z=4$). **Do not evaluate the integral.**

15. (14 points) Let $\vec{F}(x,y) = (y+2x+1)\vec{i}+x\vec{j} =< y+2x+1, x >$ be given.

   (a) First show that this vector field is a conservative vector field.

   (b) Then find a **potential** function for the given vector field.

   (c) Let C be the quarter of the circle $x^2+y^2=1$ in the first quadrant from $(1,0)$ to $(0,1)$. Use your above answers to evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$.

16. (12 points) Let $\vec{F}(x,y) = xy^2\vec{i}+2x^2y\vec{j} =< xy^2, 2x^2y >$. Use **Green's Theorem** to evaluate the line integral $\int_C \vec{F} \cdot d\,\vec{r}$, where C is the triangle with vertices at $(0,0)$, $(2,0)$, and $(0,2)$ oriented in the clockwise sense.