

Exam 1 Outline (Motivating Questions)

9.1: Functions of several variables and 3-Dimensional Space

- What is a function of several variables? What do we mean by the domain of a function of several variables?
- How do we find the distance between two points in \mathbb{R}^3 ?
- What is the equation of a sphere in \mathbb{R}^3 ?
- What is a trace of a function of two variables? What does a trace tell us about a function?
- What is a level curve of a function of two variables? What does a level curve tell us about a function?

9.2: Vectors

- What is a vector?
- What does it mean for two vectors to be equal?
- How do we add two vectors together and multiply a vector by a scalar?
- How do we determine the magnitude of a vector? What is a unit vector, and how do we find a unit vector in the direction of a given vector?

9.3: Dot Product

- How is the dot product of two vectors defined and what geometric information does it tell us?
- How can we tell if two vectors in \mathbb{R}^n are perpendicular?
- How do we find the projection of one vector onto another?

9.4: Cross Product

- How and when is the cross product of two vectors defined?
- What geometric information does the cross product provide?

9.5: Lines and planes in space

- How are lines in \mathbb{R}^3 similar to and different from lines in \mathbb{R}^2 ?
- What is the role that vectors play in representing equations of lines, particularly in \mathbb{R}^3 ?
- How can we think of a plane as a set of points determined by a point and a vector?
- How do we find the equation of a plane through three given non-collinear points?

10.1: Limits

- What do we mean by the limit of a function $f(x, y)$ of two variables at a point (a, b)
- What techniques can we use to show that a function of two variables does not have a limit at a point (a, b)

- What does it mean for a function $f(x, y)$ of two variables to be continuous at a point?

10.2: First-order partials

- How are the first-order partial derivatives of a function $f(x, y)$ of the independent variables x and y defined?
- Given a function f of the independent variables x and y , what do the first-order partial derivatives f_x and f_y tell us about f ?

10.3: Second-order partials

- Given a function f of two independent variables x and y , how are the second-order partial derivatives of f defined?
- What do the second-order partial derivatives f_{xx} , f_{yy} , f_{xy} , and f_{yx} of a function f tell us about the function's behavior?

10.4: Linearization: Tangent plane and differentials

- What does it mean for a function of two variables to be locally linear at a point?
- How do we find the equation of the plane tangent to a locally linear function at a point?
- What is the differential of a multivariable function of two variables and what are its uses?

10.5: Chain Rule

- What is the Chain Rule and how do we use it to find a derivative?
- How can we use a tree diagram to guide us in applying the Chain Rule?

10.6: Gradient and Directional Derivatives

- The partial derivatives of a function $f(x, y)$ tell us the rate of change of $f(x, y)$ in the direction of the coordinate axes. How can we measure the rate of change of $f(x, y)$ in other directions?
- What is the gradient of a function and what does it tell us?

10.7: Optimization

- How can we find the points at which $f(x, y)$ has a local maximum, minimum, or saddle point?
- How can we determine whether critical points of $f(x, y)$ are local maxima or minima, or saddle points?

Exam 1 Outline (Important Concepts/Formulas)

- Slice of a function
- Level set / Contours
- Scalar vs Vector
- Vector addition, multiplication of a vector by a scalar
- Norm = Magnitude = Length of a vector
- Dot product
- Dot product (cosine version)
- Cross Product (determinant form)
- Cross Product (sine version)
- Parallel and Perpendicular Vectors
- Equation of a line in 3D given two points
- Equation of a plane given normal vector and a point
- Three-point method of finding planes (aka the cross product method)
- Limits of functions of two variables
- Finding a limit along a given curve
- Finding $f_x(a, b)$ and $f_y(a, b)$ using limit definition
- Finding partial derivatives algebraically
- Interpretations of first-order partials in terms of increasing/decreasing
- Computing second-order partials
- Clairaut's Theorem on the symmetry of mixed second-order partials.
- Interpretation of second-order partials
- Tangent plane and the Linearization of a function
- Differential of a function
- Compute new value of a function given old value and information about the differential
- Tree diagrams and the chain rule
- Use the chain rule to write down a derivative
- Directional derivatives: definition and interpretation
- Gradient: compute, plot, use
- Gradient and directional derivative and how they're related
- Critical points: definition, how to find
- Types of critical points, how to classify them
- Second derivative test, discriminant